Discrétisation des Equations de Navier-Stokes par élements spectraux combinés avec un développement de Fourier azimutal; transition à la turbulence en géometries azimutales

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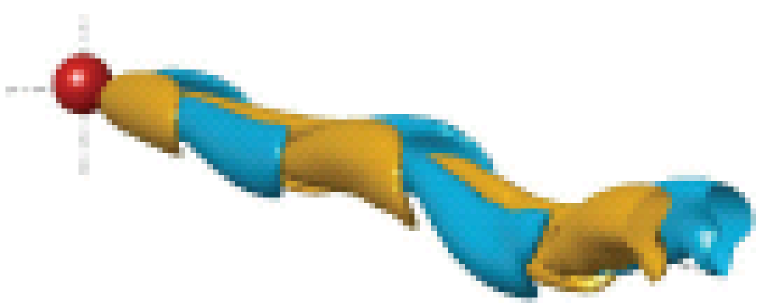
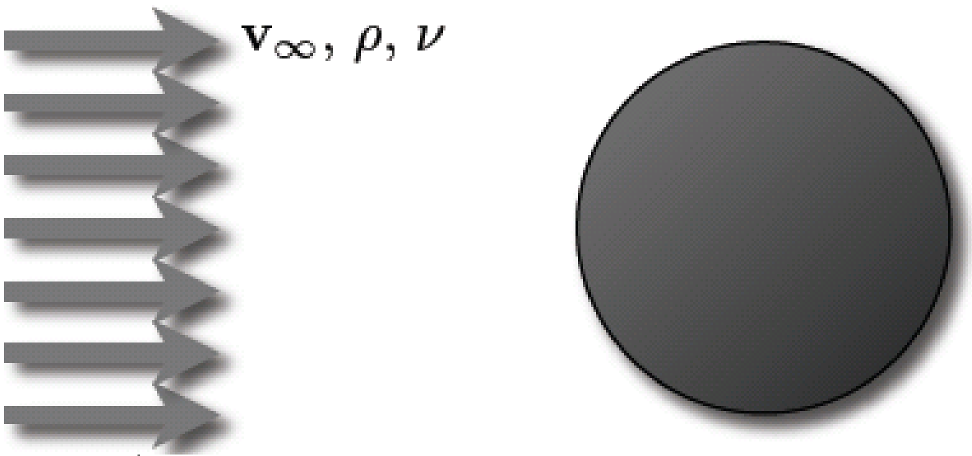


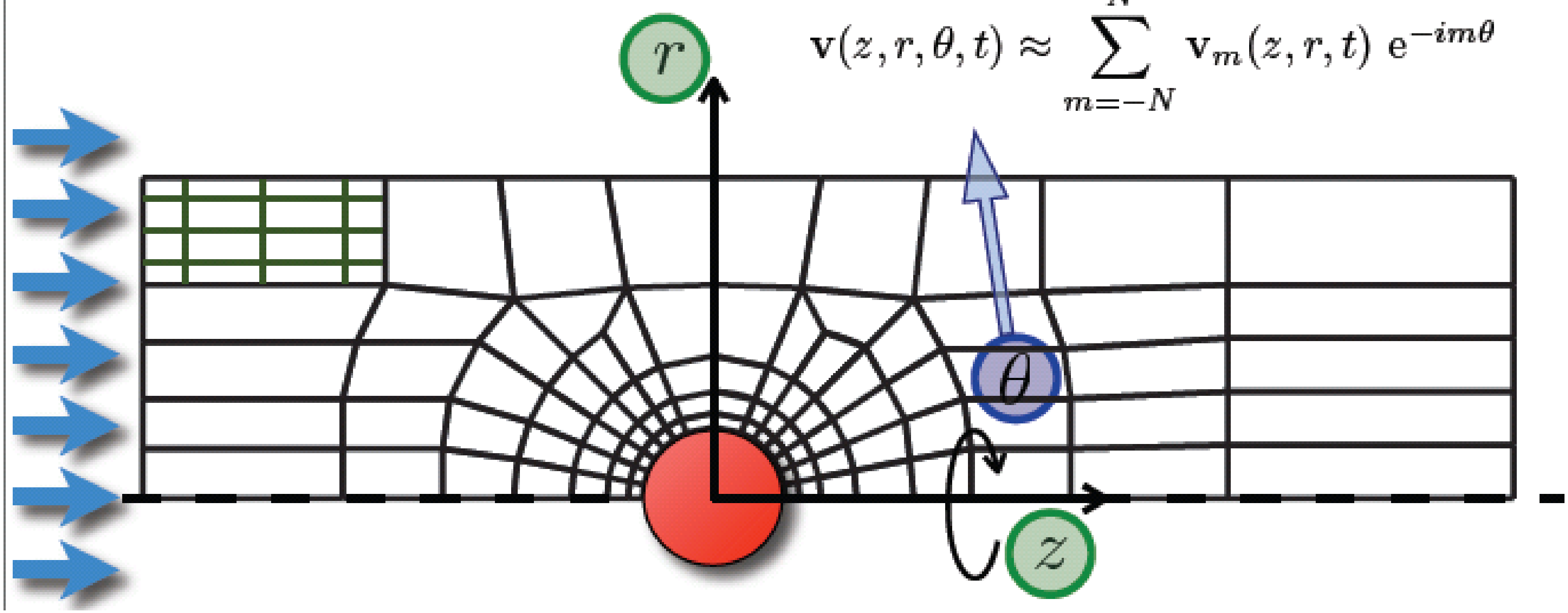
Azimuthal Fourier decomposition

Sphere wake

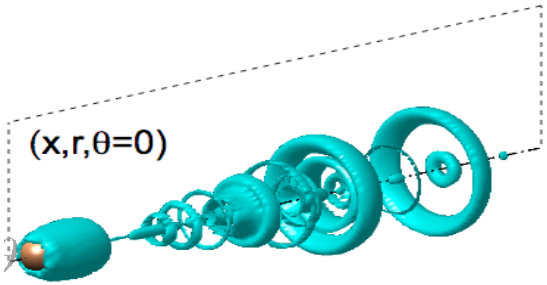


$$\mathrm{Re} = \frac{v_\infty d}{\nu}$$

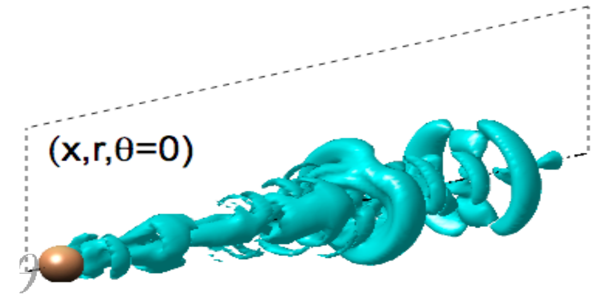
$$\mathbf{v}(z,r,\theta,t) \approx \sum_{m=-N}^{N} \mathbf{v}_m(z,r,t)\ \mathrm{e}^{-im\theta}$$



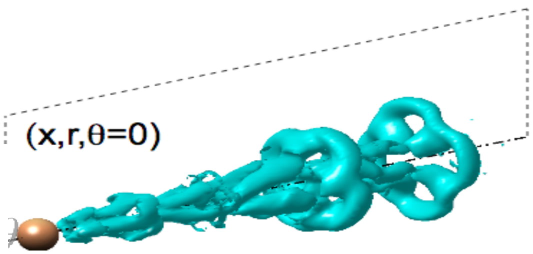
Example – superimposed modes 0 … 3
individual modes



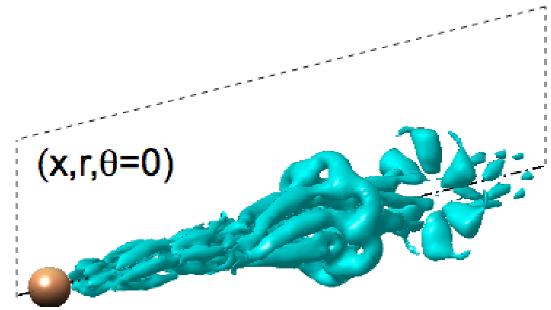
MODE 0



MODE 1

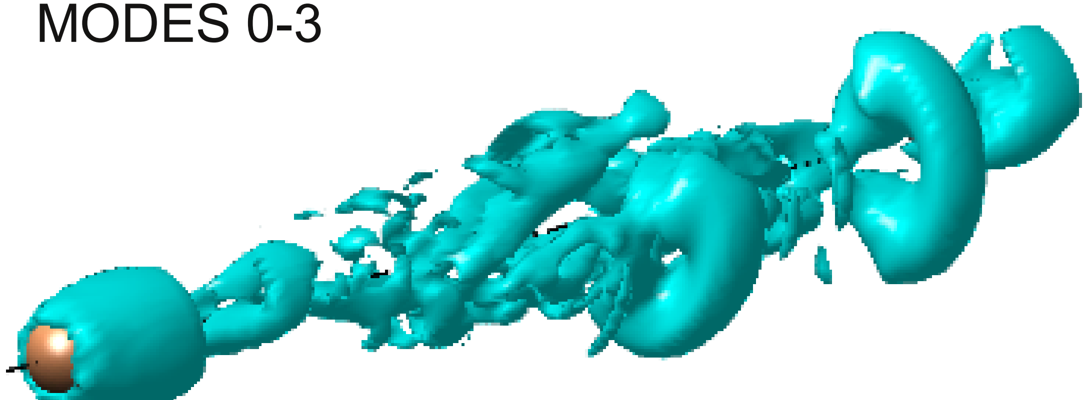


MODE 2

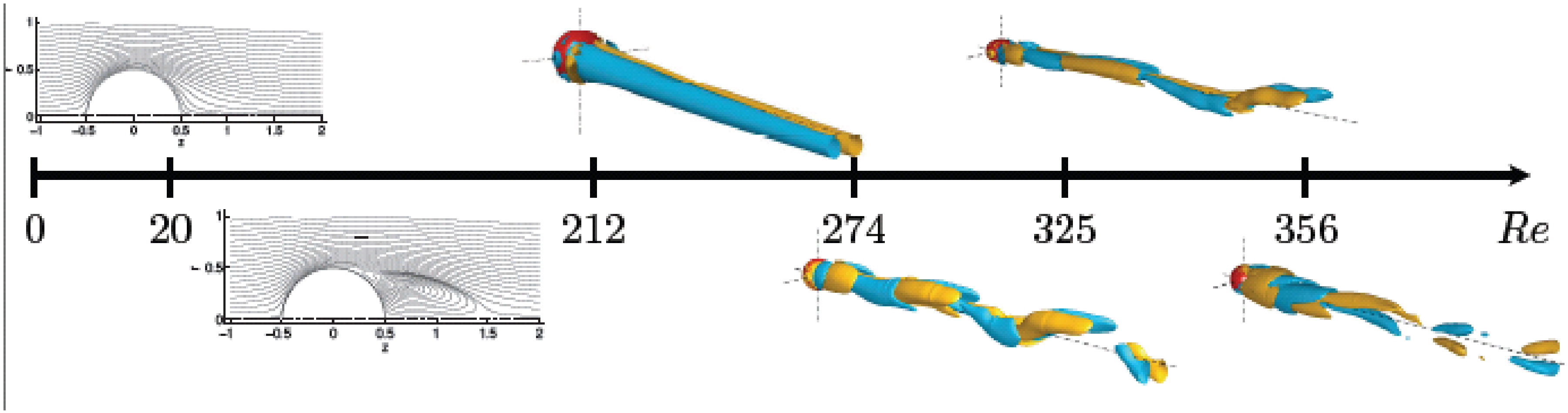


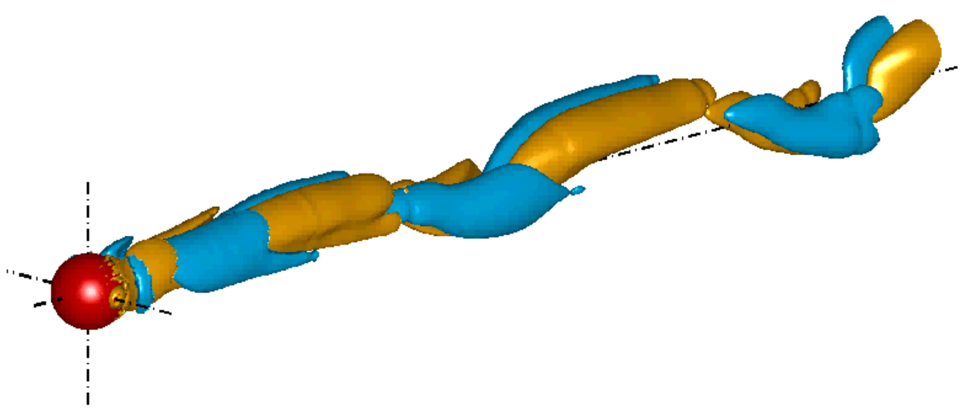
MODE 3

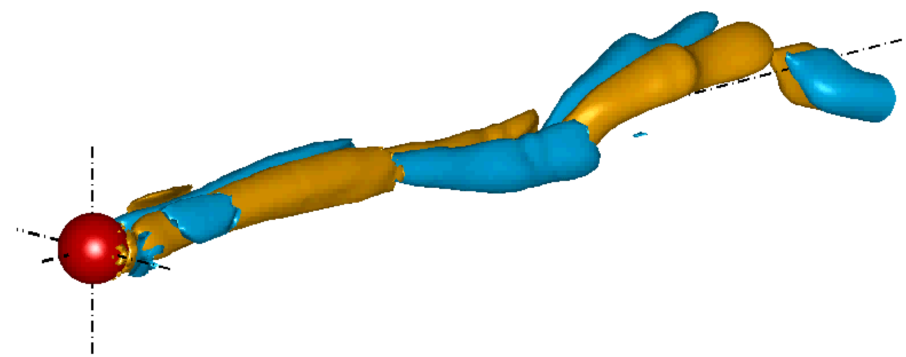
MODES 0-3



Transition scenario







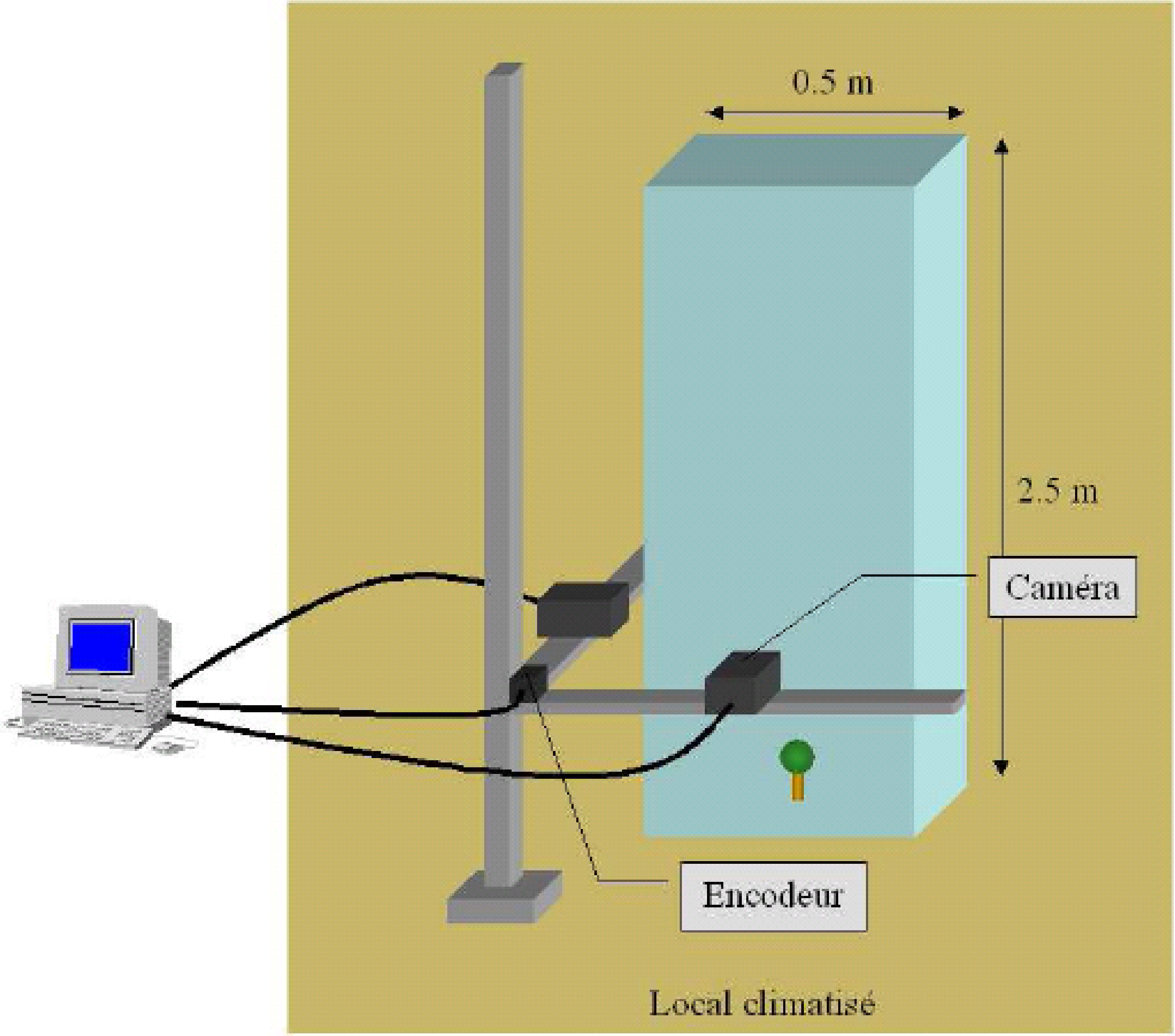
Numerical performance :

1 period : 1h CPU on DEC XP1000 (Johnson & Patel, 24 hrs on Silicon Graphics Power Challenge)

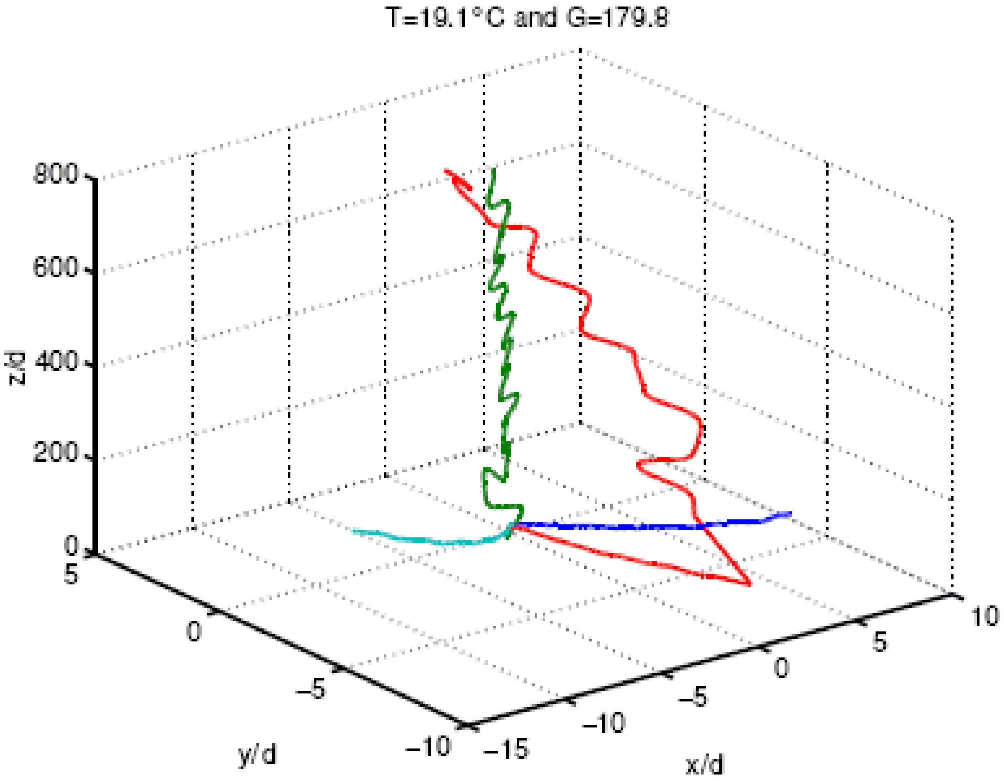
CFL criterion $\Rightarrow$ very short time step (2000/period)

CPU per time step : 80% CG pressure solver

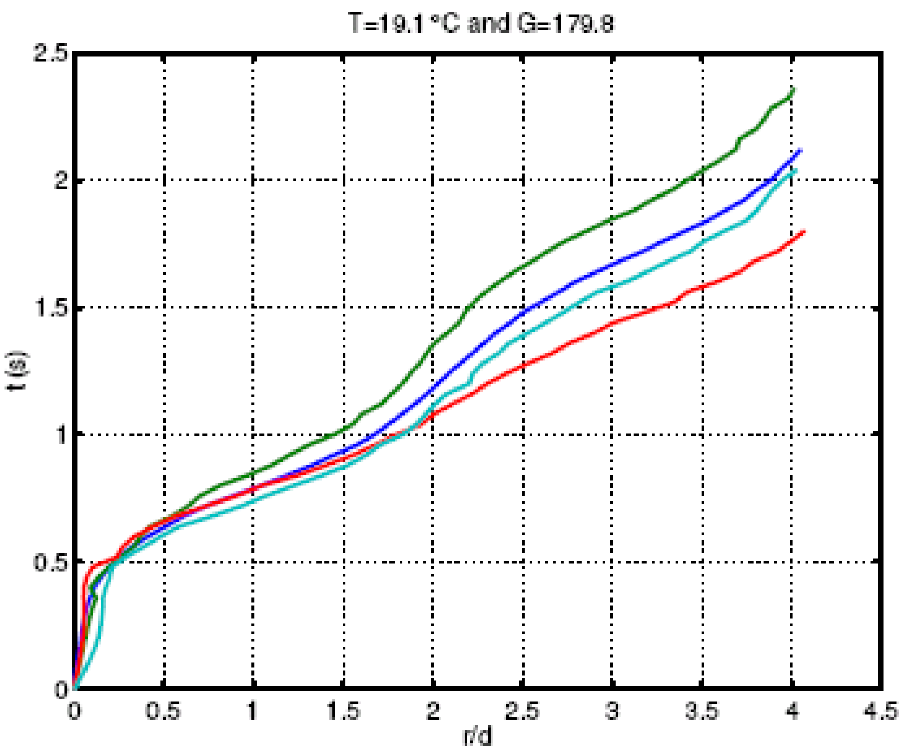
TRANSITION SCENARIO OF A FREELY MOVING SPHERE

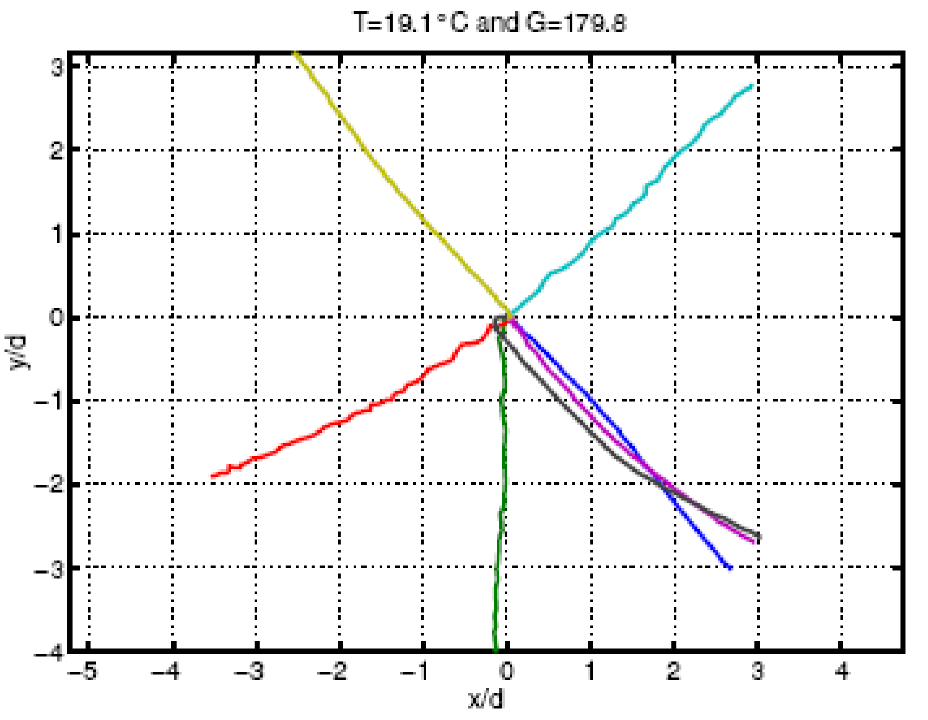


fluid density : ρ, sphere density : ρ_0



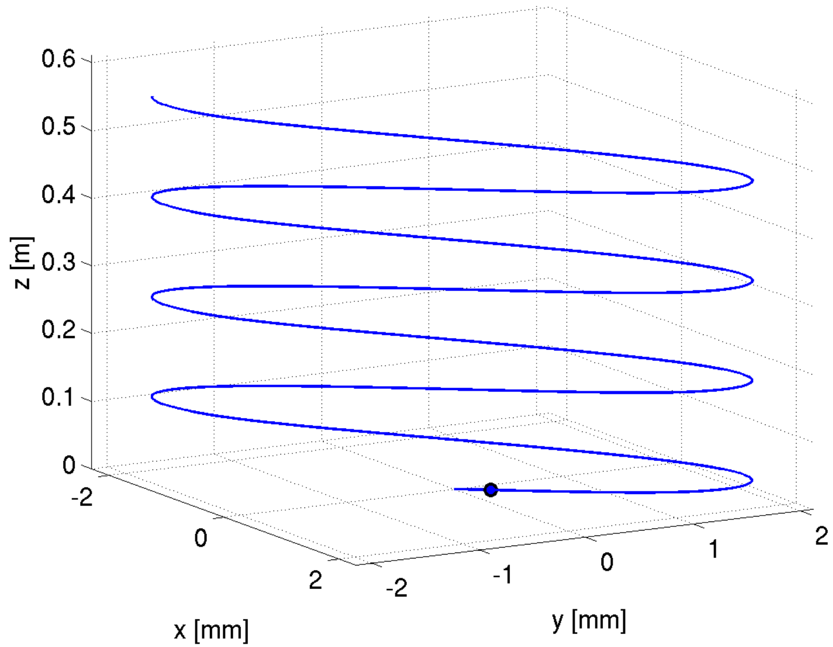
Experimetal results





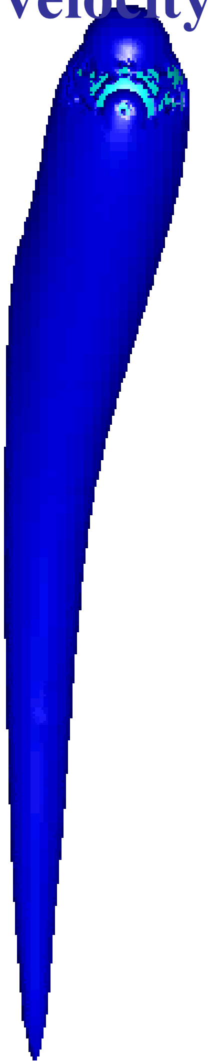
Example of result

Trajectory



Vorticity

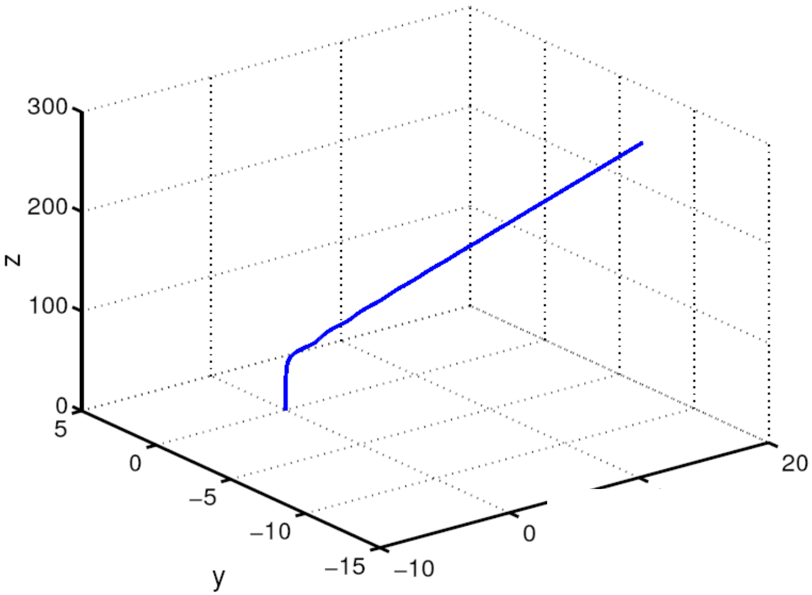
Streamwise velocity



Axisymmetry breaking : $G_{crit} \in [155, 160]$

Regular bifurcation, $m = 1$

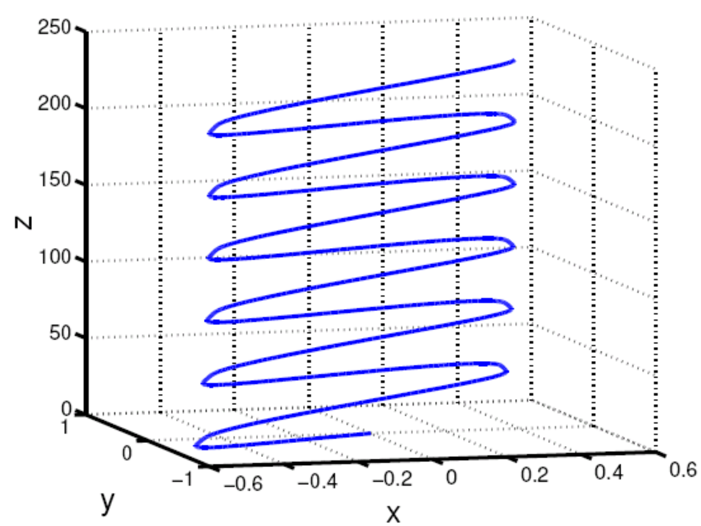
$\rho_0/\rho = 0.5,\ G = 170$



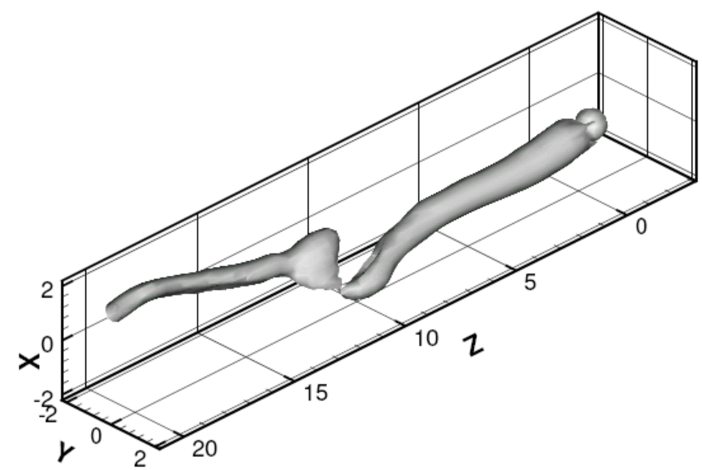
Typical regimes (attractors)

zig-zagging regime : light spheres $\rho_0/\rho < 1$

$G = 200,\ \rho_0/\rho = 0.5$

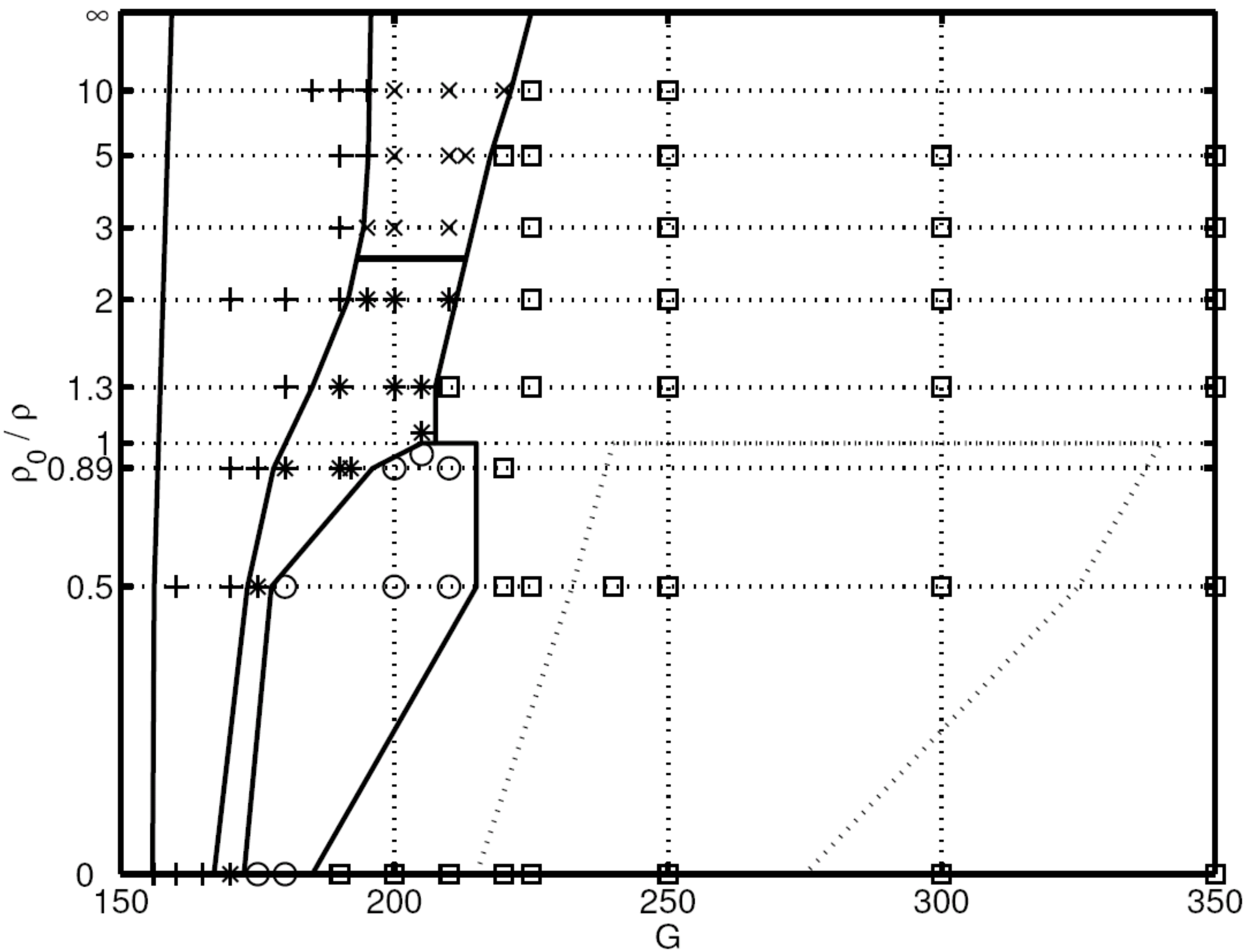


trajectory



wake : velocity iso-surface

Diagramme in the parameter space



+ : oblique steady, 'x' oblique unsteady, high frequency, '*' oblique unsteady, low frequency, 'o' zig-zag, square : chaos. (80 points)

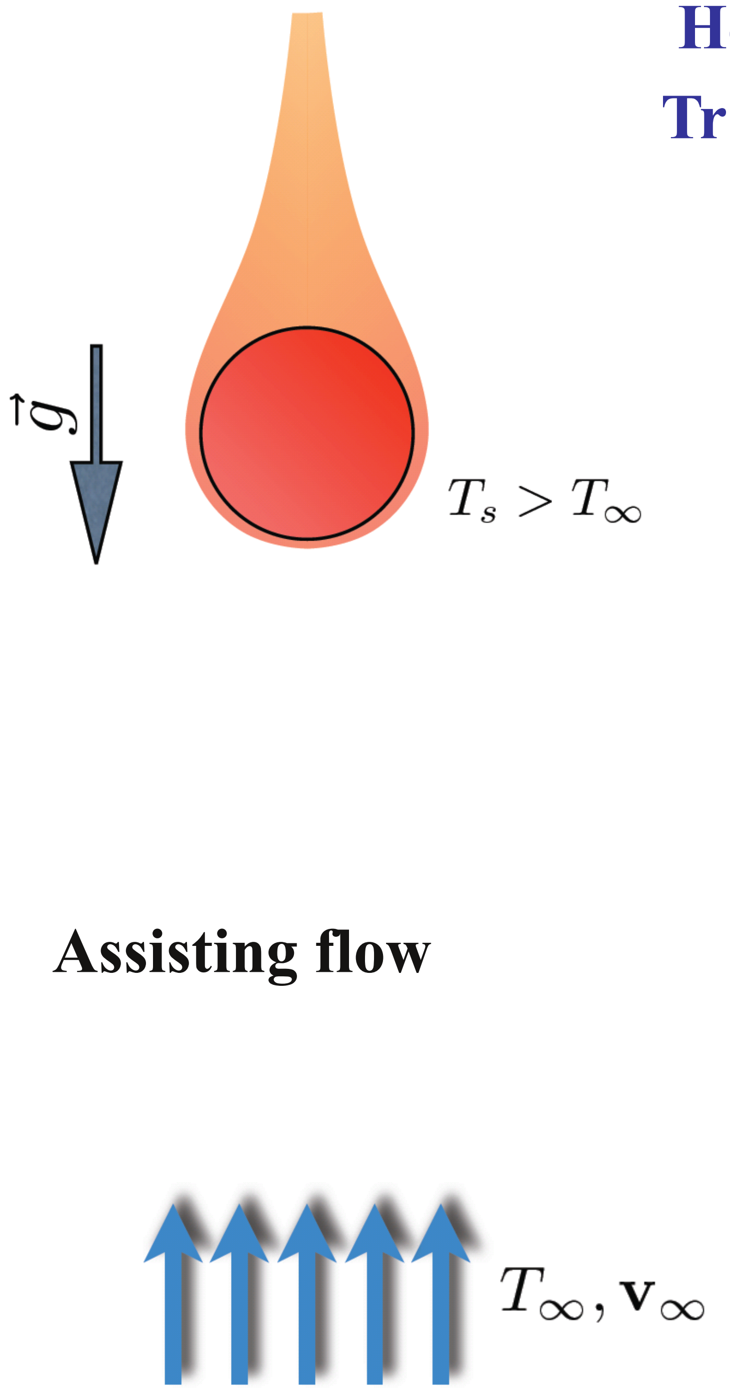
Flaws of the code:

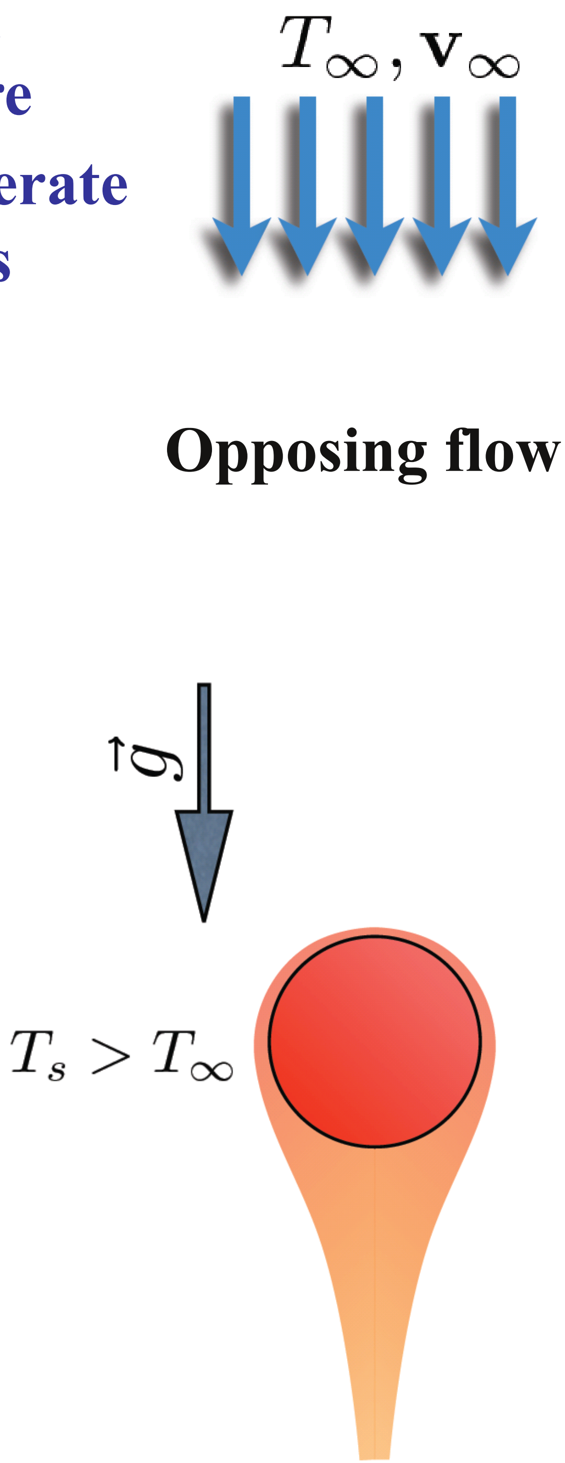
• numerical instabilities

•“imperfections” at element interfaces (non zero divergence)

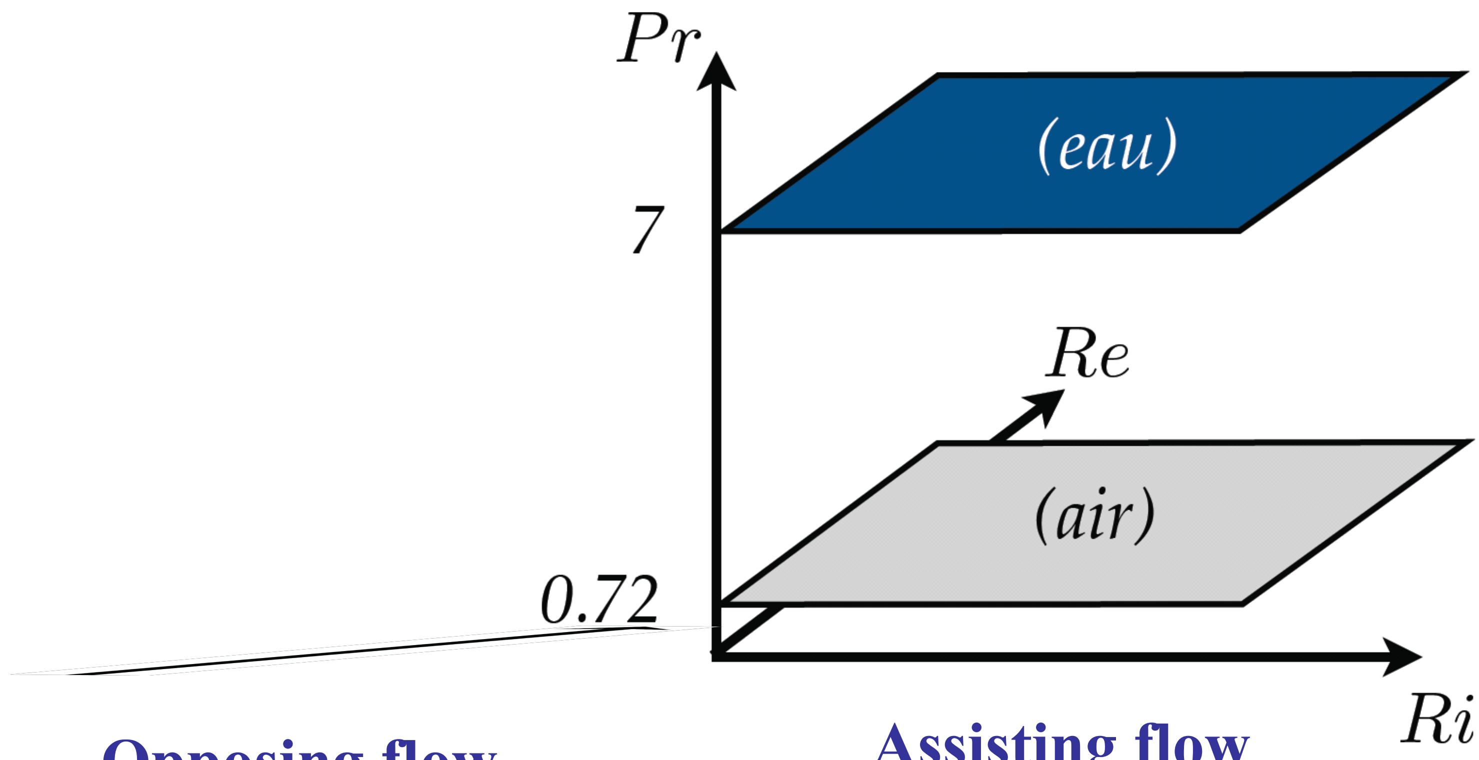
• still high computing costs (80% pressure solver)

Mixed convection
Heated fixed sphere
Transition for moderate thermal effects





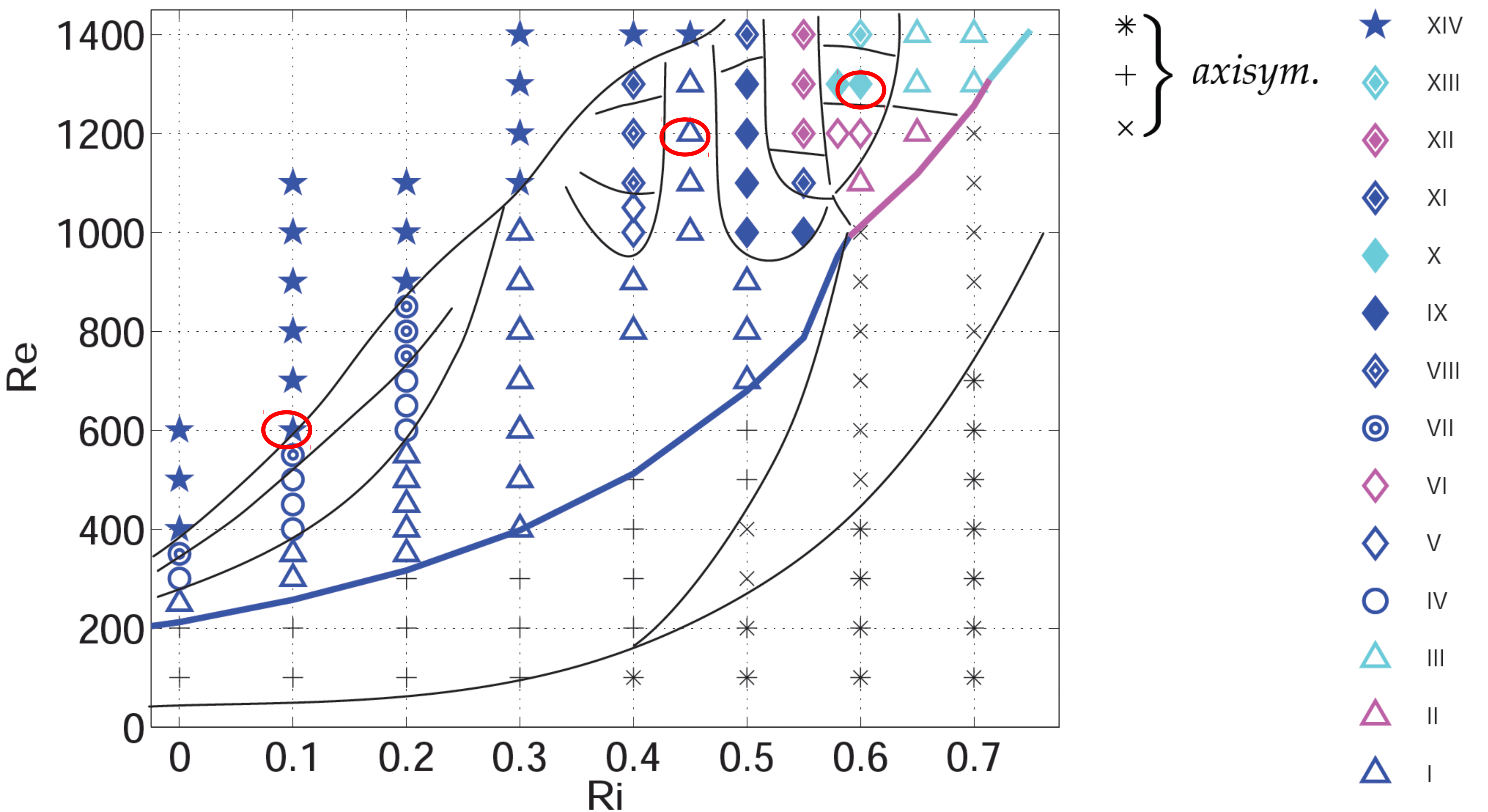
Three parameter space

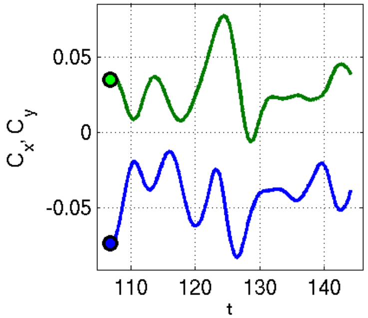


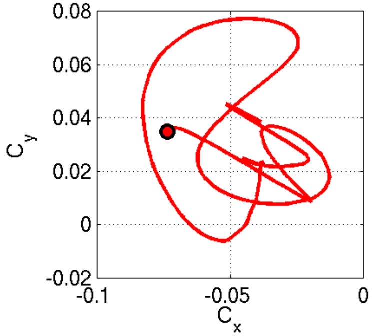
Opposing flow

Assisting flow

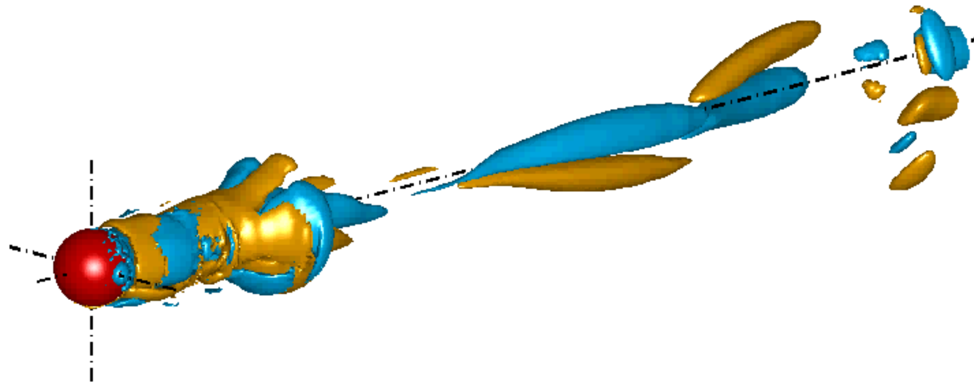
State diagram Pr=0.72, Ri>0



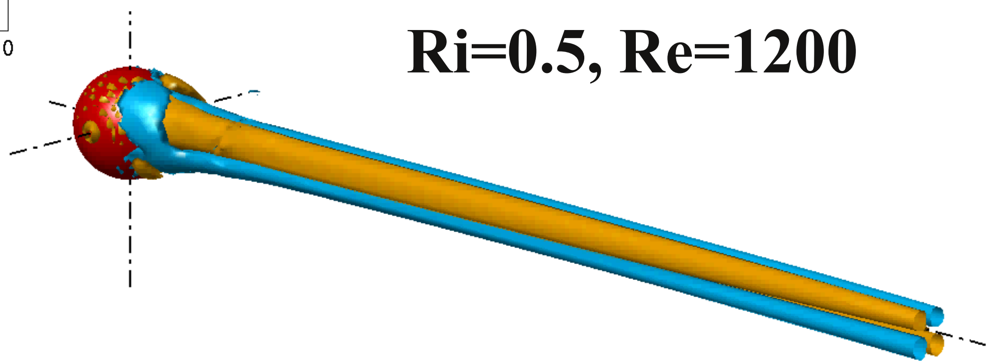


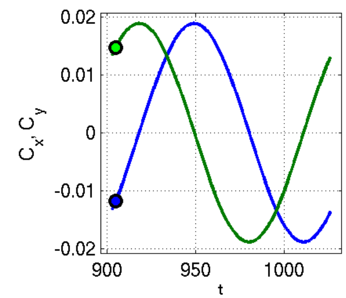


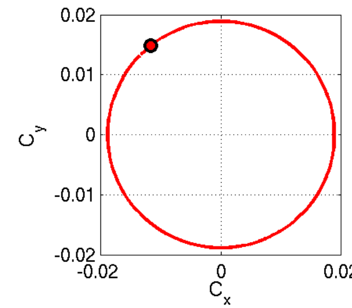
Ri=0.1, Re=600

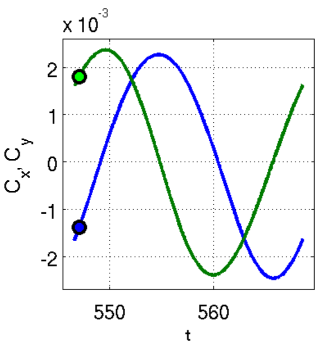


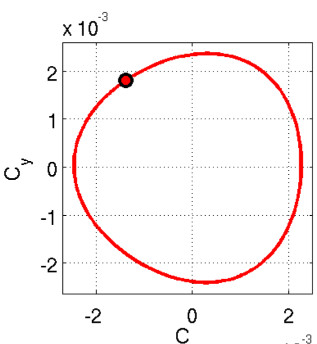
Ri=0.5, Re=1200

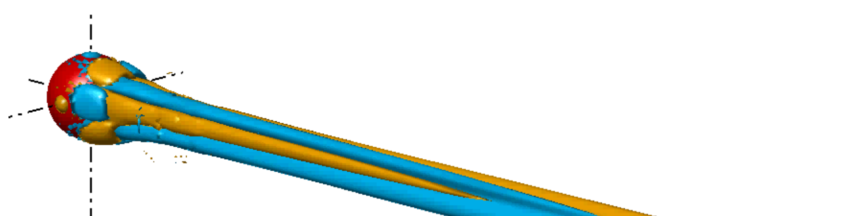






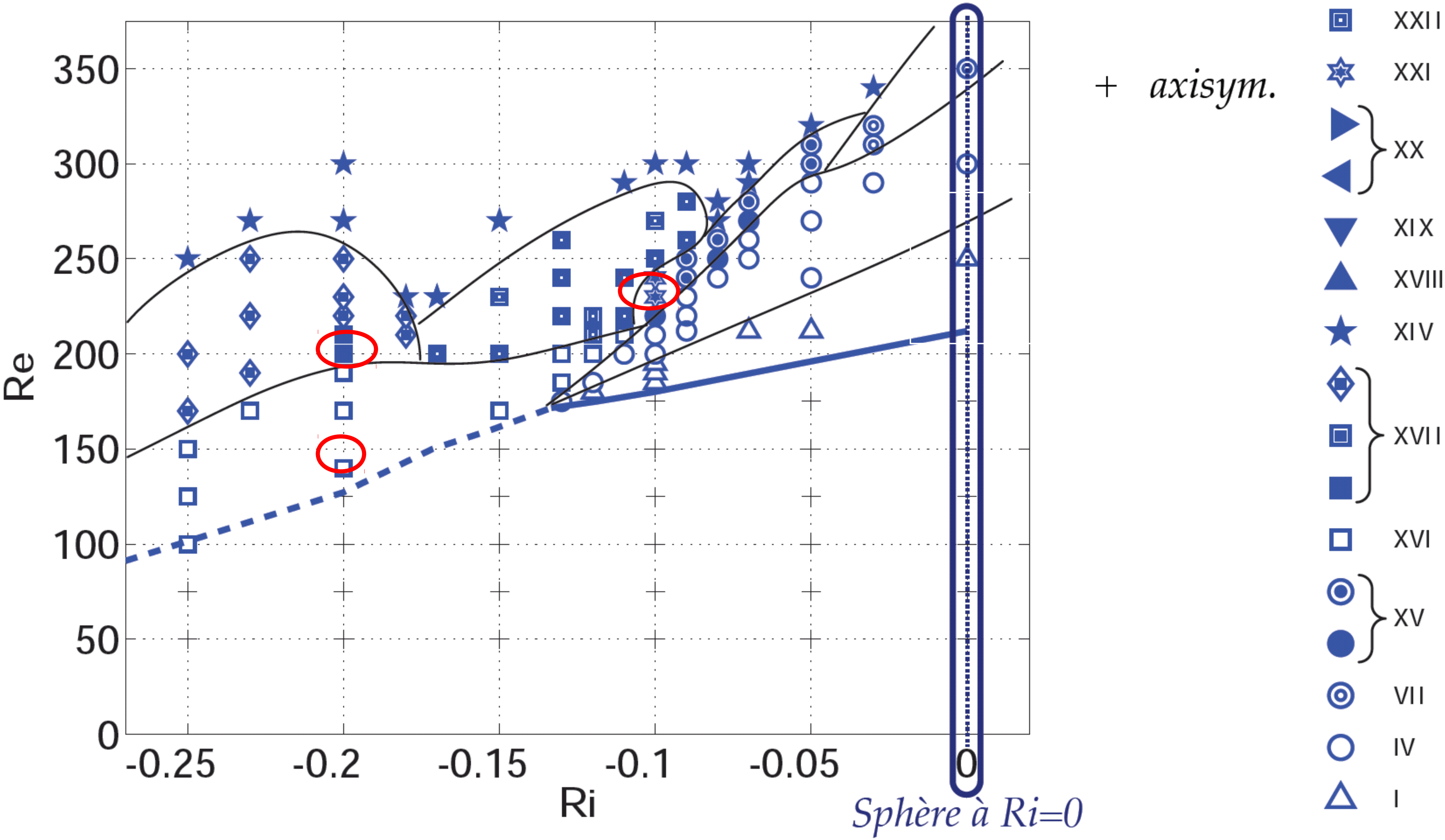


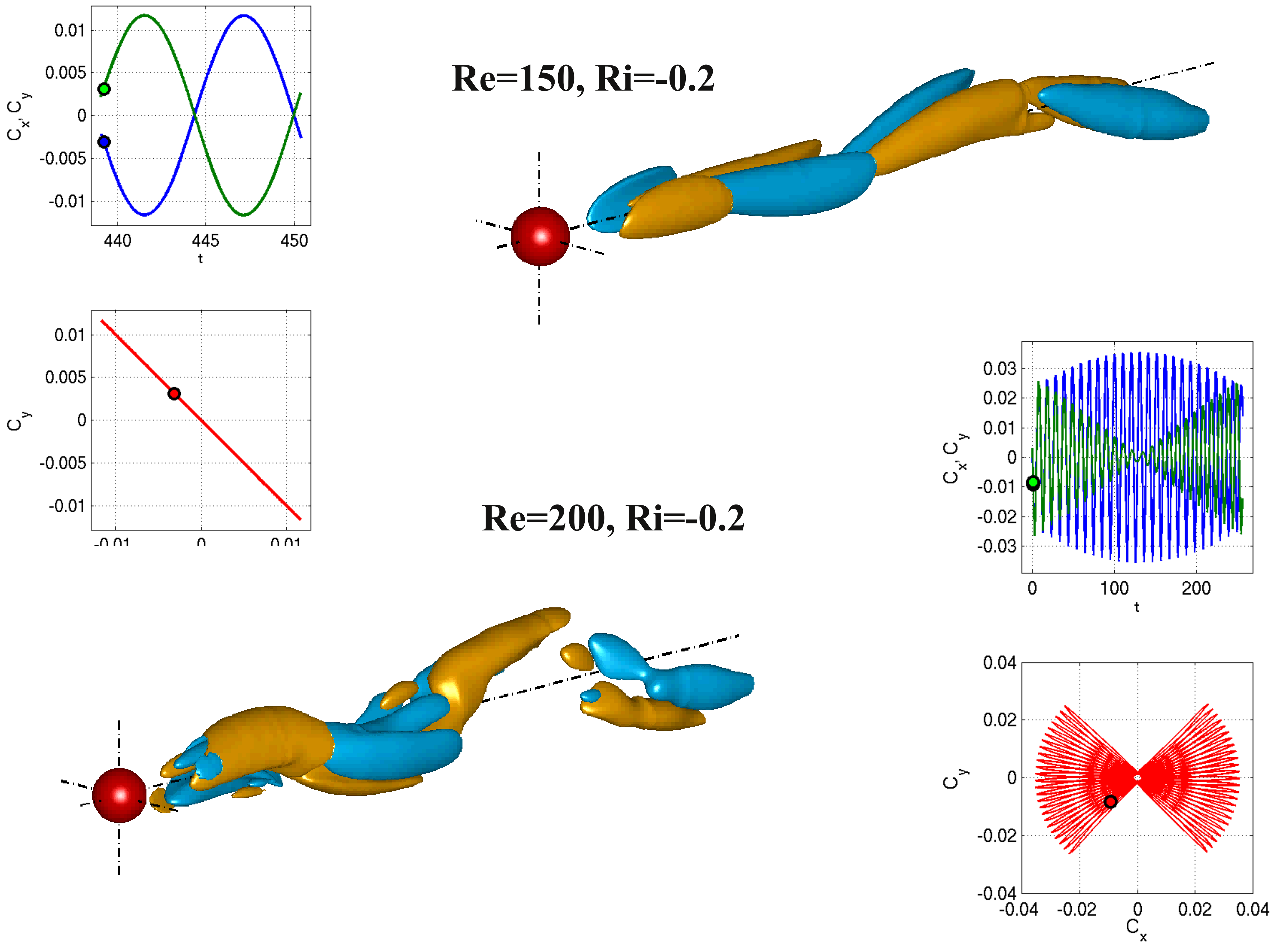




Ri=0.6, Re=1300

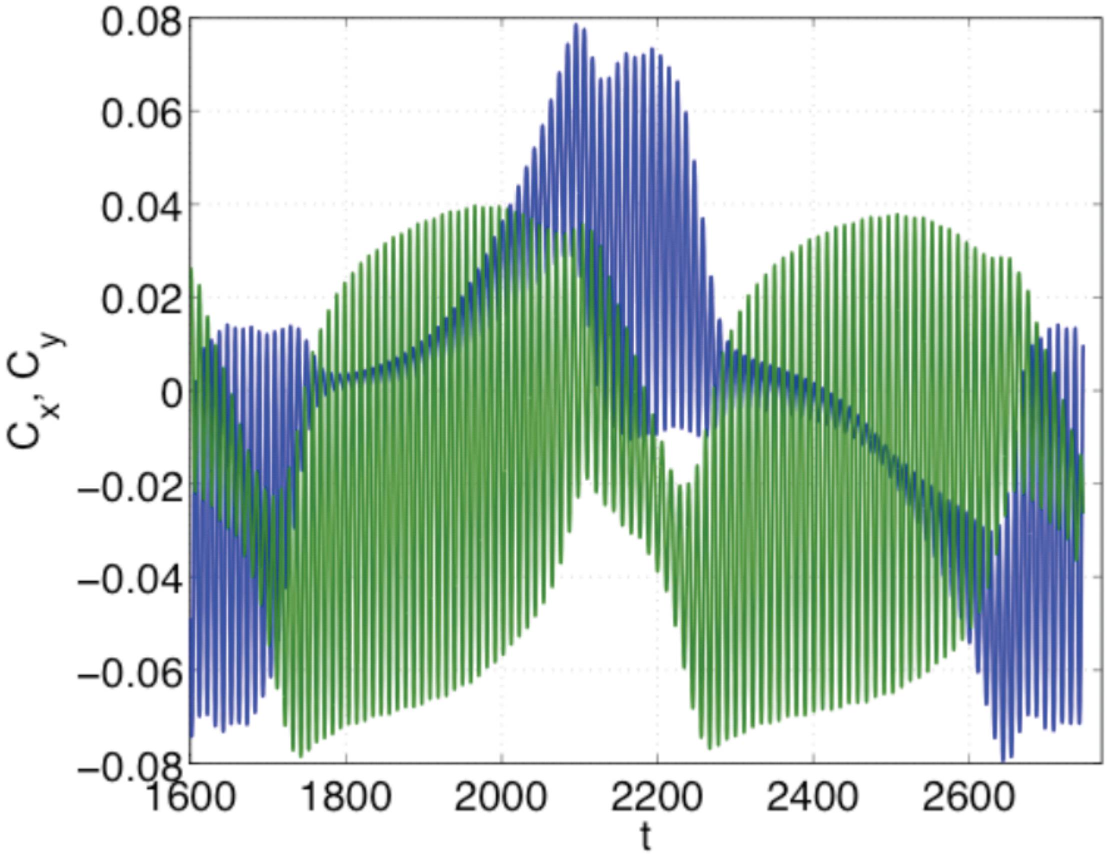
State diagram Pr=0.72, Ri<0



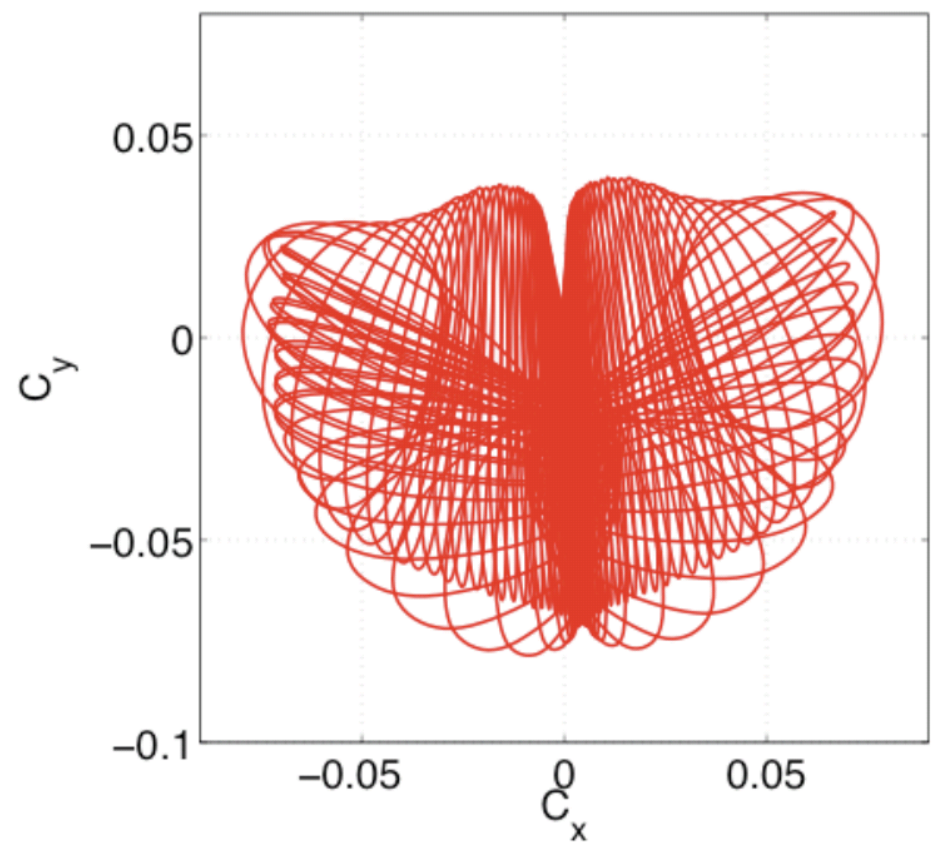


Re=150, Ri=-0.2
Re=200, Ri=-0.2
C_x, C_y
t
C_y
C_x

Re=220, Ri=-0.1



0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
−0.08
1600
1800
2000
2200
2400
2600
t
C_x, C_y

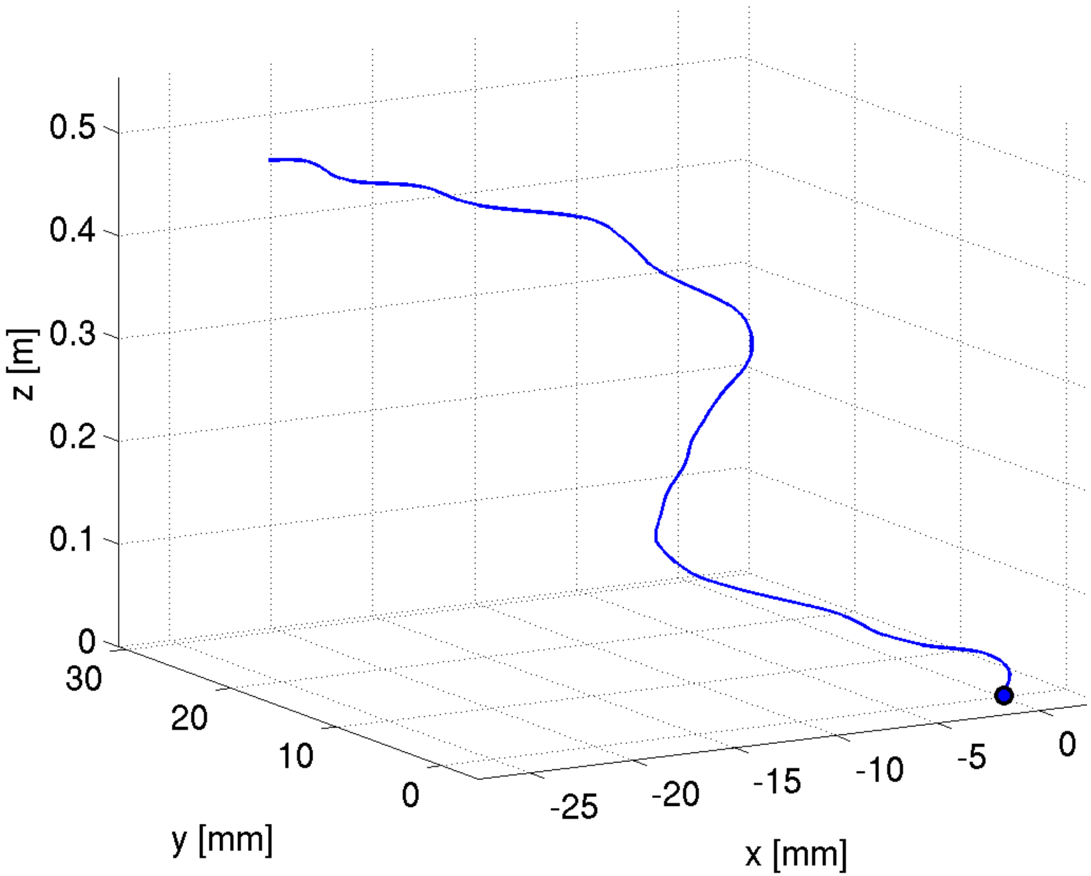


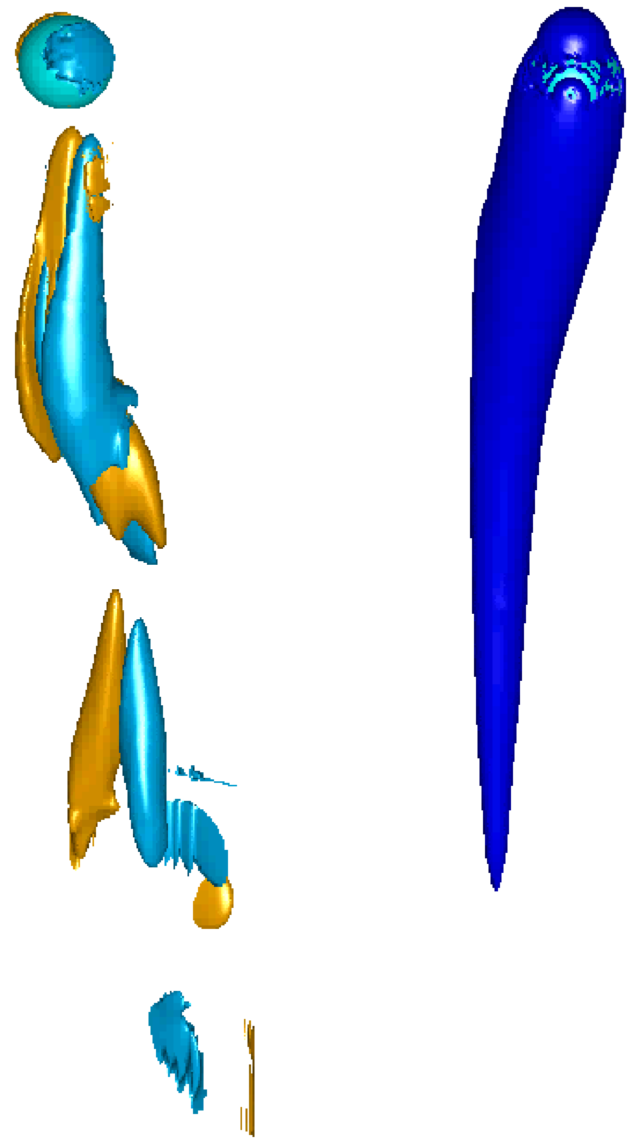
0.05
0
−0.05
−0.1
−0.05
0
0.05
C_x
C_y

Free sphere with and without fusion
(ice sphere in water)
Boussinesq not valid

T_{water} =20°C

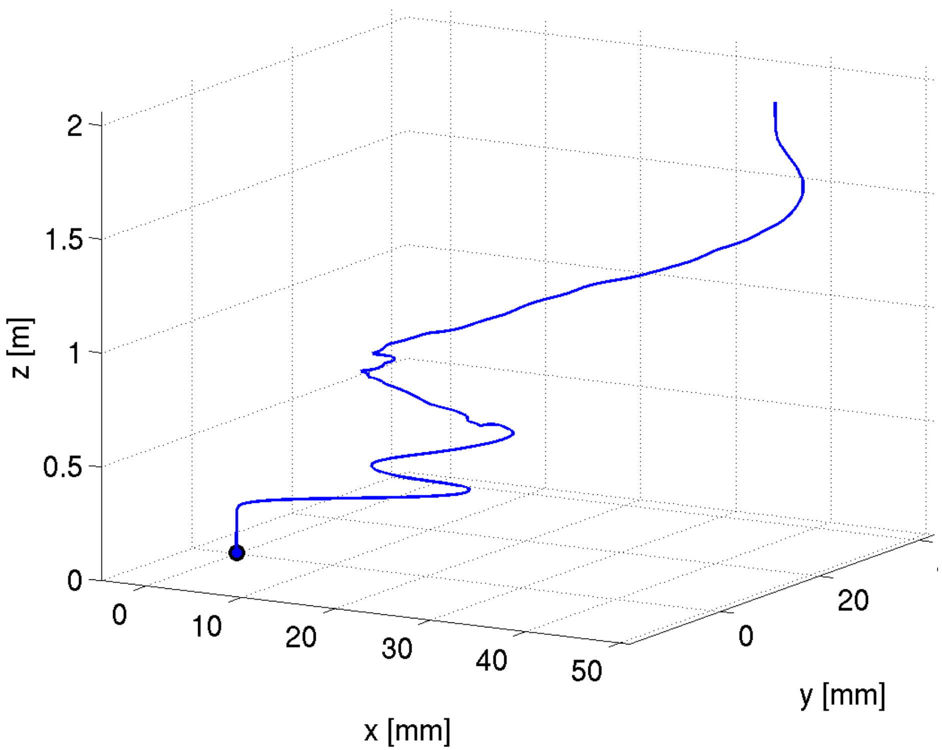
without fusion, G=200, d=4.38 mm

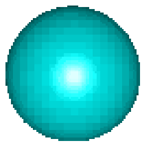


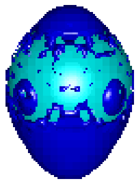


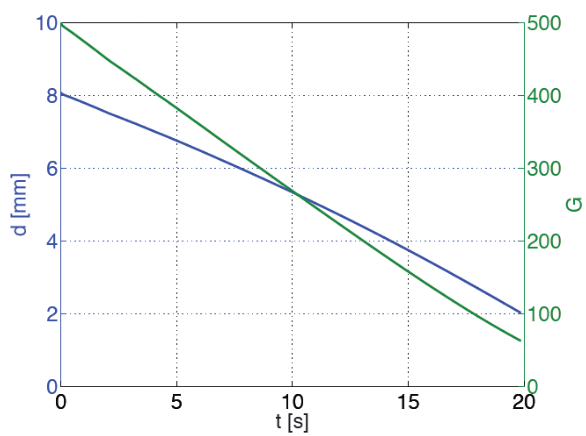
with (spherical) fusion, initial G=500, d=8.07 mm

T_{water} =20°C

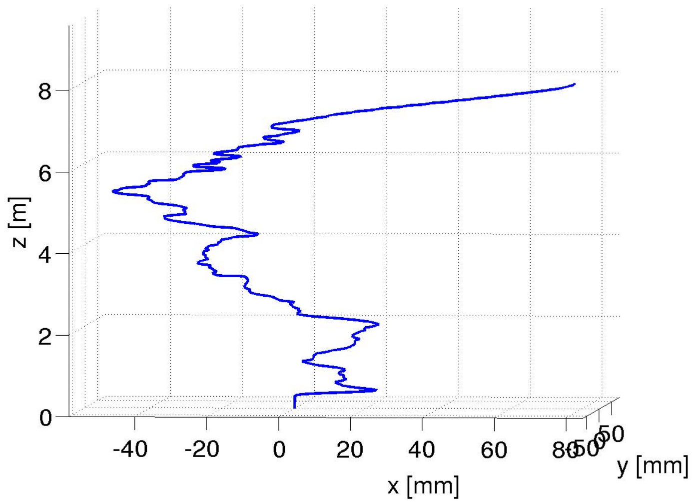








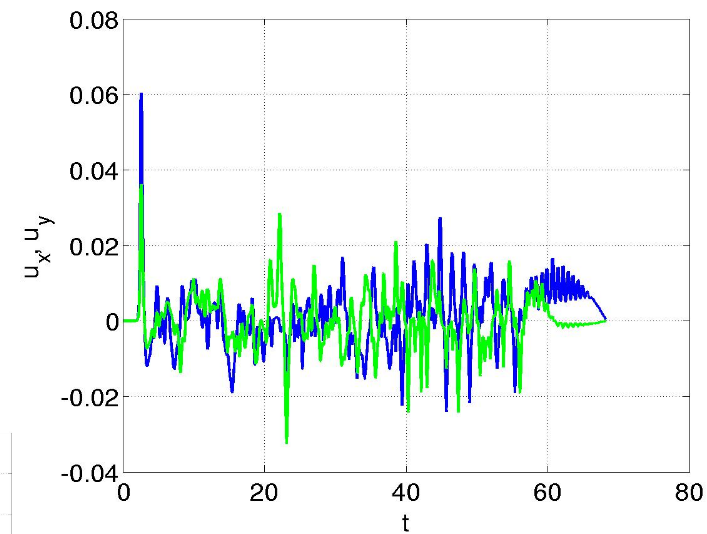
T_{water} =4°C with (spherical) fusion, initial G=500, d=9.52 mm

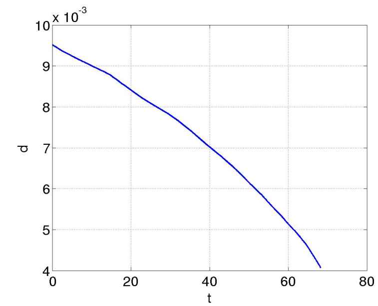


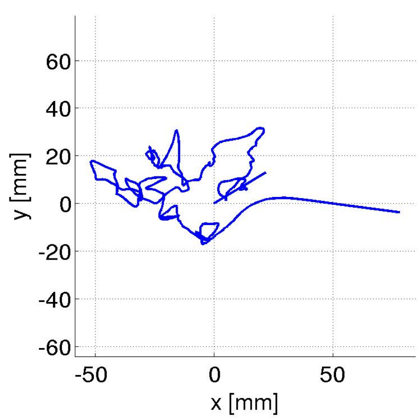
d_i=9.52 mm

d_f=4.1 mm

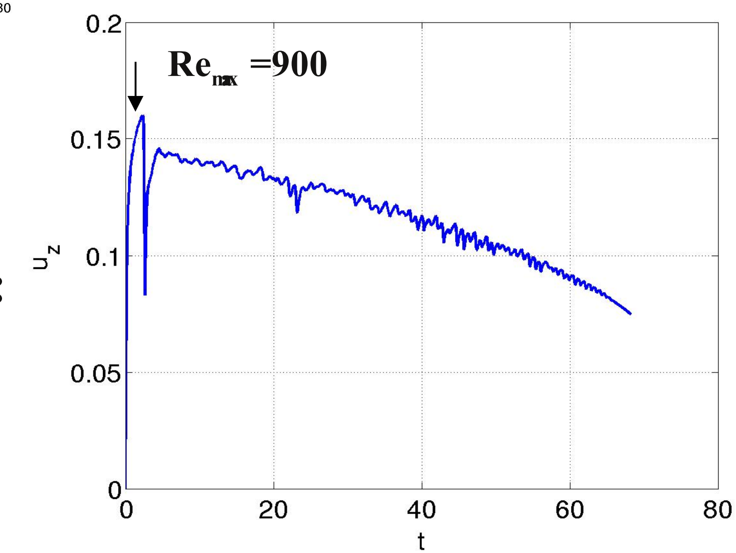
G_f=140





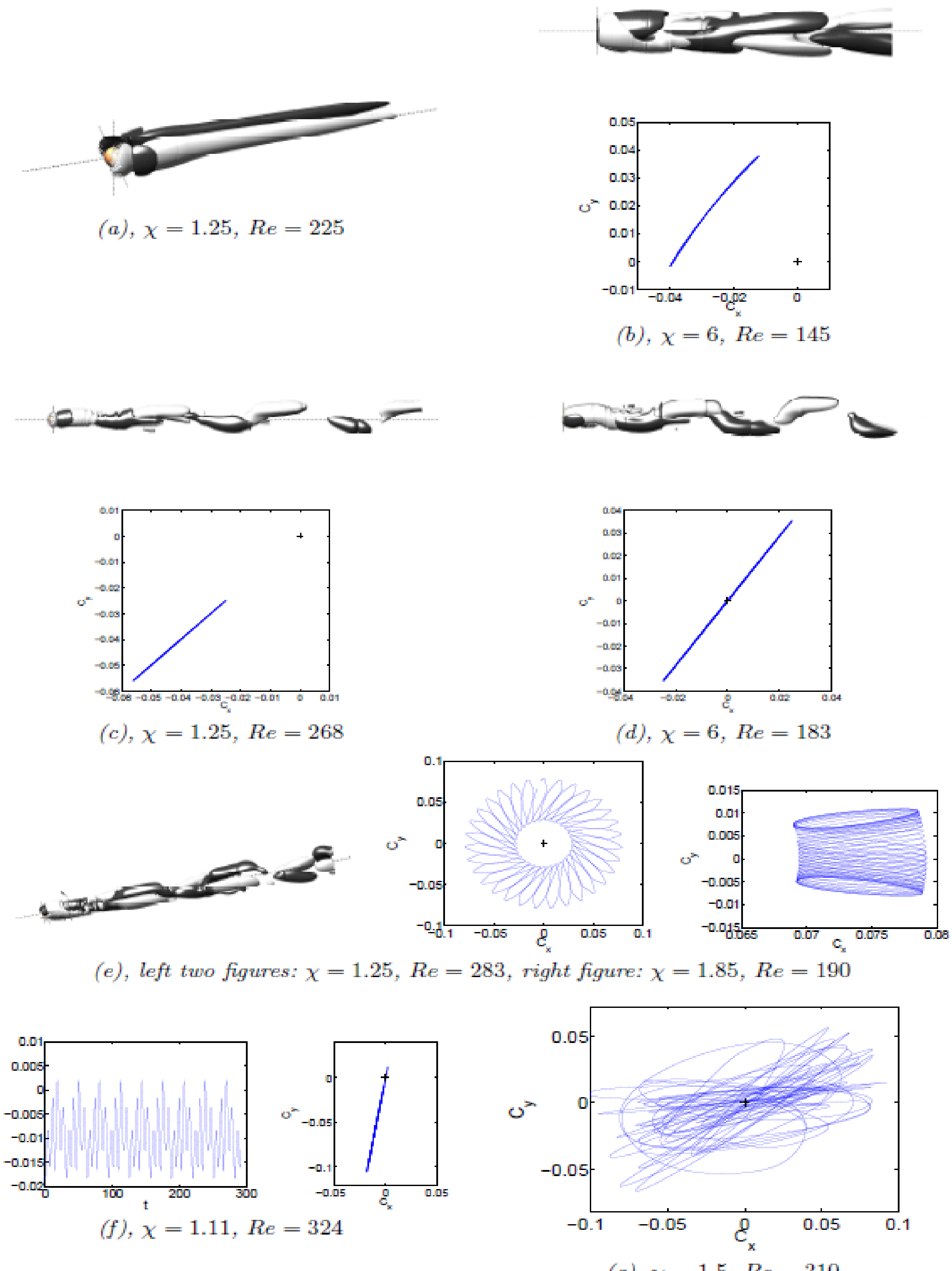


2 M time steps
start: June 25
results presented:
December 9
(still running)



Transition in the wake of disks, spheroids, flat cylinders

FIGURE 1. Spectral element discretization of the computational domain of an oblate spheroid of χ=2.



(a), $\chi = 1.25$, $Re = 225$

(b), $\chi = 6$, $Re = 145$

(c), $\chi = 1.25$, $Re = 268$

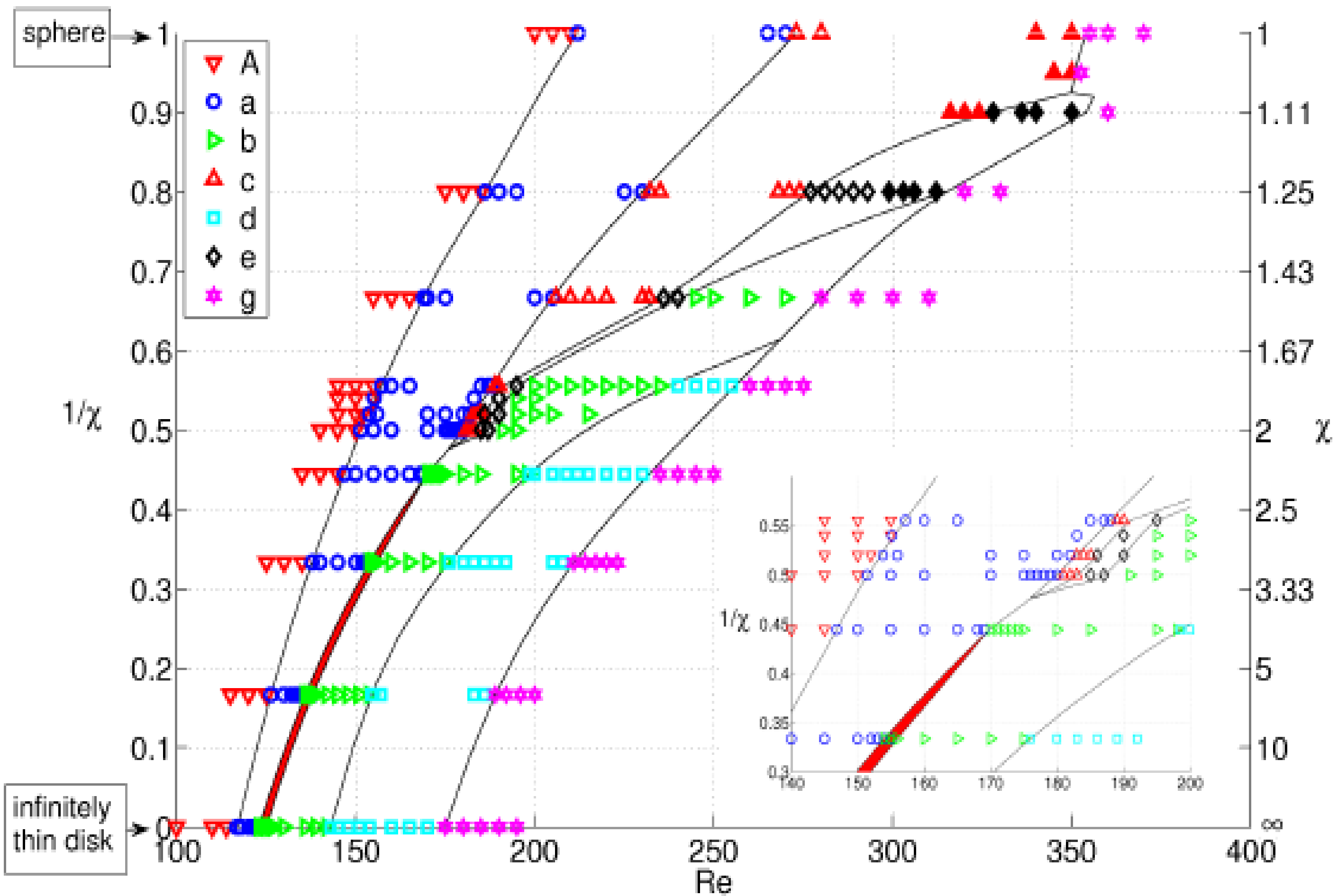
(d), $\chi = 6$, $Re = 183$

(e), left two figures: $\chi = 1.25$, $Re = 283$, right figure: $\chi = 1.85$, $Re = 190$

(f), $\chi = 1.11$, $Re = 324$

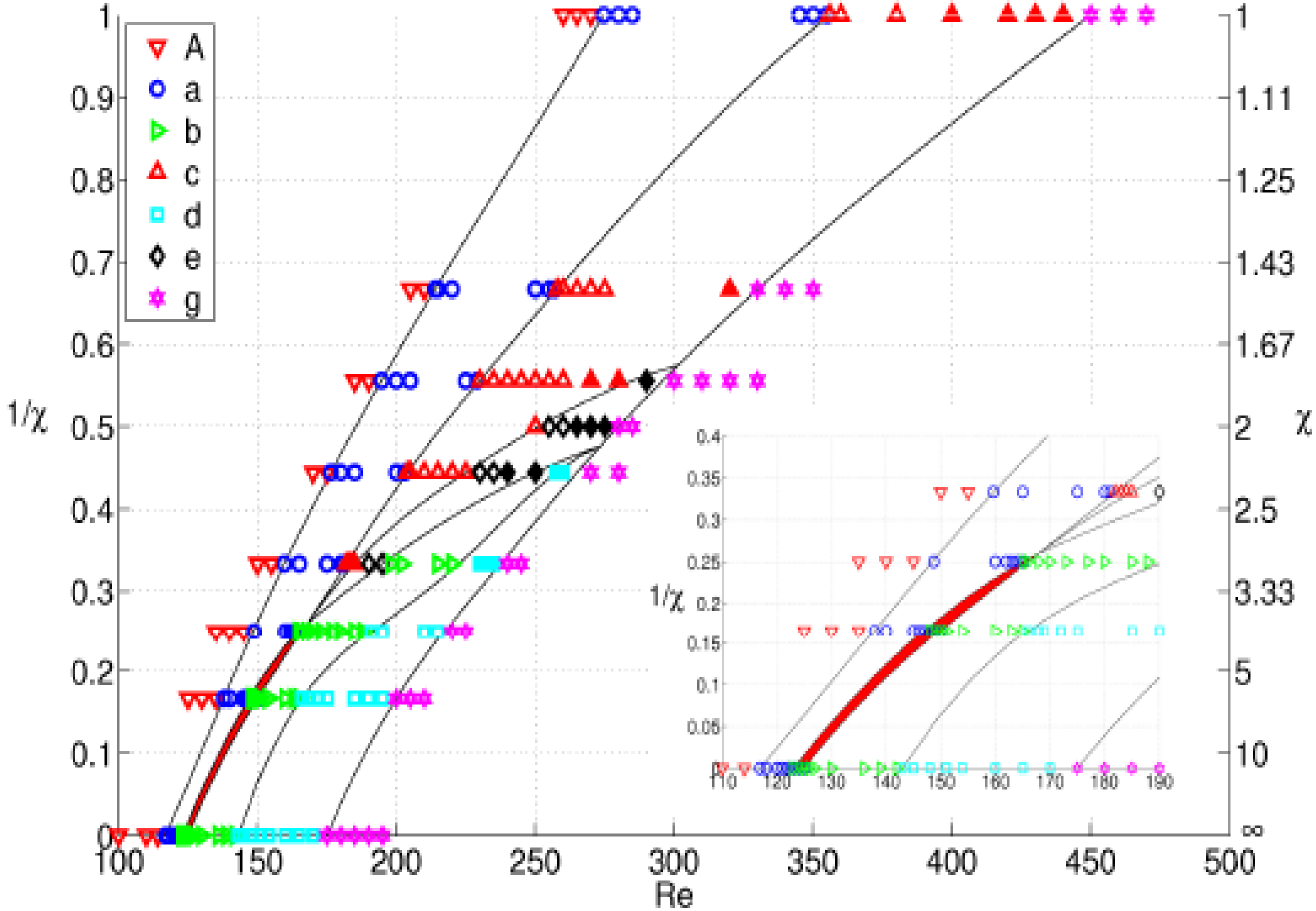
(g), $\chi = 1.5$, $Re = 310$

Spheroids



sphere
infinitely thin disk
A
a
b
c
d
e
g
1/χ
χ
Re
100
150
200
250
300
350
400
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.11
1.25
1.43
1.67
2
2.5
3.33
5
10
∞

Flat cylinders



Ordre de priorités:

1) Beaucoup de calculs indépendants (études paramétriques)

2) Rapidité d'exécution (à faible coût)